XLIII WORKSHOP ON GEOMETRIC METHODS IN PHYSICS

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LIST OF ABSTRACTS

Plenary lectures

1. **Dietrich BURDE —** *University of Vienna, Austria*

   Crystallographic actions on Lie groups and post-Lie algebra structures

   Crystallographic groups and crystallographic actions have a long history, starting with the famous list by Hilbert in 1900 asking about Euclidean crystallographic groups. Since then many generalizations have been studied, among them affine and nil-affine crystallographic actions. It turns out that these actions can be understood on the level of Lie-algebraic structures, in particular of pre- and post-Lie algebra structures on Lie algebras. We present several results on the existence and classification of such structures, for semisimple and reductive Lie groups, but also for solvable and nilpotent ones.

2. **Javier DE LUCAS —** *University of Warsaw, Poland*

   Introduction to k-contact geometry and applications

   k-contact geometry is a generalisation of contact geometry that naturally extends its scope to the geometric formulation of field theories, while preserving and enriching many of the characteristic features of classical contact geometry.

   This talk aims to introduce the fundamentals of k-contact geometry and to illustrate its potential applications in classical mechanics, differential equations, and field theories. In particular, applications to thermodynamical relativistic hydrodynamical systems, integrable systems, Dirac-type equations, k-contact Lagrangian equations, control systems, characteristics of Lie symmetries, and conditional symmetries will be briefly discussed.

3. **Yuji HIROTA —** *Azabu University, Japan*

   Lie algebroid symmetry with bundle-valued n-plectic geometry

   The theory of momentum maps is central in studying spaces with symmetry and has developed in various directions. Following the discovery of symmetries arising from Lie algebroids, the need to understand the classical notion of momentum maps in a unified framework has further increased. The notion of homotopy momentum sections is one of such frameworks.

   In this talk, we introduce the fundamentals of homotopy momentum sections and geometry associated with it, known as bundle-valued n-plectic geometry. Moreover, we shall provide the relationship with quaternionic Kähler momentum maps, and shall briefly discuss a reduction theorem analogous to the Marsden–Weinstein reduction in symplectic geometry.

4. **Shahn MAJID —** *Queen Mary University of London, United Kingdom*

   Quantum and classical geodesic flows on a black-hole background

   I describe a recent formulation with Beggs of generally covariant quantum mechanics, which in the Schrödinger picture amounts to wave functions on spacetime governed by a proper time Klein–Gordon flow. Recent work with K. Kumar extends results previously found on the exterior of a black-hole to inside the black hole, finding now a discrete mass spectrum of 'atomic' solutions of the Klein–Gordon equations, where the spectrum depends mildly on the regularisation of the theory by cutting off the domain to just inside the horizon. We also show with Kumar how the theory applies to classical geodesics on a black-hole background viewed as a density flow and, hypothetically, as an underlying wave-function amplitude flow. We show that two 'bump' wave functions of opposite sign can collide to form a dipole with a different density profile from two density bumps merging when they collide. If time, I will also describe some results with Beggs for Klein–Gordon flows on FLRW cosmological backgrounds.

5. **Francesco TOPPAN** — *Centro Brasileiro de Pesquisas Físicas, Brazil*

   A colored look at anyons and parastatistics

   Paraparticles beyond bosons/fermions belong to two main classes: the anyons, which are exchanged under the braid group and exist up to 2 space dimensions, and the paraparticles, theoretically possible in any space dimension, exchanged via the permutation group. For both classes, parabosonic and parafermionic particles can be found, with parafermions satisfying a (generalized) Pauli exclusion principle. A natural way to accommodate these parastatistics is via Rittenberg–Wyler's framework of color Lie (super)algebras graded by abelian groups. The first theoretical signature of paraparticles living in any space dimension was produced for $\mathbf{Z}_2 \times \mathbf{Z}_2$-graded parafermions. More general grading groups, such as $\mathbf{Z}_3 \times \mathbf{Z}_3$, allow to accommodate anyons in the RW's framework. A state of the art of theoretical and experimental advances is briefly recalled.

6. **Alejandro VARELA** — *Instituto Argentino de Matemática (Alberto P. Calderón) CONICET, Argentina*

   The Riemann sphere of operators on Hilbert space

   Given a Hilbert space H, consider $\tilde{a} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in B(H \oplus H)$ (with $a_{ij} \in B(H)$). The Riemann sphere of $B(H \oplus H)$ is the unitary orbit

   $$\mathcal{R} = \left\{ \tilde{u} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tilde{u}^{-1} : \tilde{u} \in \mathcal{U}(H \oplus H) \right\},$$

   where $\mathcal{U}(H \oplus H)$ denotes the unitary operators in $B(H \oplus H)$. We present the elements of $\mathcal{R}$, which are given by projections $P_{\mathrm{Gr}(T)}$ onto the graphs $\mathrm{Gr}(T) = \{(h, Th) : h \in H\}$ of densely defined closed operators T. We discuss general results and concrete examples, such as the case of the unbounded operator $-i\frac{d}{dx}$. In this context, we exhibit geodesics of projections onto graphs, natural approximations of unbounded operators by bounded operators, and Jacobi fields related to Fredholm operators.

7. **Karol ŻYCZKOWSKI** — *Jagiellonian University, Poland*

   A rich garden of metrics in the space of quantum states

Contributed lectures

1. **Suvrajit BHATTACHARJEE** — *University of Białystok, Poland*

   Crystallization and K-theoretical aspects of the quantum twistor bundle

   In this talk, I will report on an ongoing work with Sophie Emma Zegers. I will focus on the K-theoretical aspects of the quantum twistor bundle, which is an example of a quantum fiber bundle with a quantum homogeneous space as fibers, as introduced by Brzeziński and Szymański. In a recent work, Zegers describes the K-theory group of the total space of the bundle and conjectures a relation amongst the K-theory (and K-homology) groups of the fiber, the total space and the base of the quantum twistor bundle. I will describe this conjecture and how, using tools from the recently introduced notion of crystallization by Matassa–Yuncken and Laca–Neshveyev–Yamashita, we are able to settle this conjecture affirmatively.

2. **Tomasz BRZEZIŃSKI —** *University of Białystok / Swansea University, Poland / UK*

   Affunctional analysis

   In the talk I will describe an attempt to formulate basic concepts of functional analysis, such as Banach space, Banach algebra, Hilbert space, C^*-algebra etc., in the category of affine spaces. Joint work with B.K. Kwaśniewski.

3. **Goce CHADZITASKOS —** *Czech Technical University in Prague, Czech Republic*

   The space proper-time manifold

   We propose an alternative interpretation of the space-time structure in which proper time plays the role of an embedding parameter of a three-dimensional spatial manifold into a four-dimensional space. In this framework, coordinate time is interpreted as the length of a trajectory in the higher-dimensional manifold. We show that the principle of least action naturally follows from Lorentz invariance and discuss implications for cosmology, classical mechanics, and wave phenomena.

4. **Jan CIEŚLIŃSKI —** *University of Białystok, Poland*

   Standard and nonstandard exponential functions on time scales, including discrete and q-calculus cases

   A time scale is a closed domain on which we can define functions and dynamical systems. The extreme cases are differential equations (the continuum case) and difference equations (the discrete case). Considering an arbitrary time scale allows for a unified treatment of the continuous and discrete cases, q-analogues, and all intermediate cases, in particular fractal ones. Within time-scale calculus, the term exponential function usually denotes the delta exponential function or, less frequently, the nabla exponential function (both are related to well-known q-analogues). Unfortunately, these standard exponential functions fail to map the imaginary axis onto the unit circle, leading to rather poor properties of the associated trigonometric functions which are typically neither periodic nor bounded. We present alternative classes of exponential functions leading to trigonometric functions with significantly better properties, including the Pythagorean identity. These constructions are related to symmetric Padé approximants. The simplest, yet most important, example is the Cayley exponential function. An important motivation for this approach comes from the relationship between dynamical systems on discrete time scales and numerical integration methods. In this setting, first-order delta and nabla equations correspond to the explicit and implicit Euler schemes. respectively, while other numerical schemes applied to linear equations can produce other types of discrete exponential functions. In the continuous setting, exact solutions of many equations are naturally expressed in terms of exponential or trigonometric functions. We present analogous examples of exact solutions for equations on time scales, including linear equations with constant coefficients, the logistic equation and the sine-Gordon equation. In the last case both the delta exponential function and the Cayley exponential function prove to be useful in deriving the exact soliton solution.

5. **Tomasz CZYŻYCKI —** *University of Białystok, Poland*

   Periodic vs. chaotic orbits for iterated physical systems

   Many interesting physical phenomena are described by iterated systems, especially by difference equations of the first and second order. In linear case the solution and its behavior is more predictable than in nonlinear case where the chaotic effects appear. In this talk we focus on homographic difference equations applied in physics and properties of their solutions. We show chaotic and periodic orbits of various order, fixed points and stability analysis of solutions in several cases, theoretically and numerically demonstrated.

6. **David J. FERNÁNDEZ C.** — *CINVESTAV, Mexico*

Confluent algorithm and spectral manipulation in quantum mechanics

I will present the confluent second-order supersymmetric quantum mechanics as a simple algorithm for manipulating spectra of quantum mechanical systems. The integral as well as differential version of this method will be discussed. I will illustrate how the technique works through one simple example.

7. **Manuel GADELLA** — *University of Valladolid, Spain*

Some applications of the Birman–Schwinger operator in quantum mechanics

The energy-dependent Birman–Schwinger operator helps to solve, at least approximately, the eigenvalue problem associated with the Schrödinger equation, under the condition of having a negative definite potential (as for instance the Coulomb potential). Then, the eigenvalues of the Schrödinger equation are those values of the energy for which the Birman-Schwinger operator has an eigenvalue equal to one. For those cases in which the Birman–Schwinger operator is trace class, these solutions coincide with the zeroes of the Fredholm generalized determinant.

In the present talk, we shall discuss two examples of application of the method.

8. **Jean-Pierre GAZEAU** — *University of Białystok / Université Paris Cité, Poland*

A structural rule behind octonionic geometry and Hopf-type constructions

I present a unified perspective on two seemingly distinct constructions arising in non-associative algebra and geometric representation theory, both governed by a simple structural rule that we denote by RLLR / LRRL. This rule emerges naturally in the manipulation of Cayley–Dickson algebras and, independently, in the organization of higher Hopf-type fibrations and their octonionic extensions.

This approach relies on a mnemonic and operational framework that clarifies how non-associativity can be consistently controlled through ordered multiplication patterns. In particular, I show that the same combinatorial scheme (RLLR/LRRL) governs:

- the internal consistency of iterated Cayley–Dickson constructions,
- the structure of certain octonionic matrix representations,
- and the coherence of higher Hopf-like geometric mappings.

This observation suggests the existence of an underlying structural principle bridging algebraic and geometric aspects of octonionic frameworks. The resulting viewpoint provides a simplified and unified handling of otherwise intricate non-associative computations, and opens the way to further applications in mathematical physics, especially in contexts where exceptional structures and higher symmetries play a role.

9. **Gerald GOLDIN** — *Rutgers University, USA*

Anyonic quantum particle structures from representations of a universal kinematical group

Some interesting intermediate multiplicative phases in quantum-mechanical wave functions, associated with local rotations, arise from theoretically possible internal multipole structures of particles. Such "internal anyonic phases" are independent of the exchange statistics of the particle configurations. I will describe results in both two and three space dimensions. These new physical possibilities follow direclty from classifying the continuous unitary representations of a universal kinematical group for quantum mechanics. Results are in collaboration with Hongyi Shen at Rutgers University.

10. **Tomasz GOLIŃSKI** — *University of Białystok, Poland*

Poisson structures and Banach Lie algebroids

In finite dimension there is an equivalence between Lie algebroids and linear Poisson structures on its dual. In Banach case the situation turns out to be more complicated. Namely the Lie algebroid structure no longer leads to a Poisson bracket defined for all smooth functions on the dual. As demonstrated by F. Pelletier and P. Cabau, one can still define a kind of weak Poisson (or sub-Poisson) bracket on some subalgebra of smooth functions. However as an alternative approach one can replace the dual of the Lie algebroid by the predual and obtain stronger results. An example of a precotangent bundle is presented. The existence of queer Banach Lie algebroids will also be briefly discussed.

11. **Giorgi KHIMSHIASHVILI** — *Ilia State University, Georgia*

Topology of generalized Sklyanin algebras

The topology of Casimir levels of quadratic Poisson algebras will be discussed. In particular, the Euler characteristics of Casimir levels of the four-dimensional generalized Sklyanin algebras will be computed and the arising bifurcations of the Casimir levels will be described. Our results give an extension of the analogous results for the classical Sklyanin algebras and yield a correction of an error in the original paper of E. Sklyanin.

12. **Bartosz KWAŚNIEWSKI** — *University of Białystok, Poland*

Regular C^*-irreducible inclusions and Galois correspondence

An inclusion of C^*-subalgebra A in a C^*-algebra B is C^*-irreducible if all intermediate C^*-subalgebras C are simple (Rordam). Such inclusions arise naturally in the study of actions of groups on C^*-algebras, either as crossed products (Kishimoto, Cameron–Smith, Amrutam–Kalantar), fixed point algebras (Izumi, Mukohara) or their mixture (Echterhoff–Rordam). In this talk I will discuss C^*-irreducible inclusions which are regular in the sense of Kumjian. It follows that hey correspond to outer Fell bundles over discrete groups G, and so up to Morita-equivalence all such inclusions are given by crossed products by an outer group actions. Using a modification of Magajna's separation theorem considered recently by Kennedy–Ursu, I can give a simple proof that the intermediate C^*-subalgebras are in bijective correspondence with subgroups of G. This generalises Galois correspondence of Cameron–Smith, and improves upon results of Izumi and Mukohara in the case of abelian groups.

13. **Réamonn Ó BUACHALLA** — *Charles University, Czech Republic*

One-Cross Bundles and Noncommutative Geometric Yang–Baxter Solutions

One-cross bundles provide a new framework for constructing noncommutative geometries on quantum homogeneous spaces. The framework is modelled on classical Hermitian symmetric spaces and their quantum analogues, the cominuscule quantum flag manifolds. The differential calculus associated to a one-cross bundle admits a unique equivariant connection, which is moreover a bimodule connection. We show that the corresponding bimodule map satisfies the Yang–Baxter equation, thereby producing a new family of Yang–Baxter solutions that we term noncommutative geometric solutions. The irreducible quantum flag manifolds equipped with the Heckenberger–Kolb calculus serve as motivating examples. Time permitting, we discuss the challenges to extending this construction to the full quantum flag manifolds.

14. **Anatolij PRYKARPATSKI** — *Cracow University of Technology, Poland*

On the Gupta–Bleuler and Dirac–Fock quantization schemes revisiting

I revisited the Gupta–Bleuler and Dirac–Fock quantization schemes in quantum electrodynamics, guaranteeing the energy positivity of photon quanta via the weak Lorenz condition, imposed on the related Fock space. Having reanalyzed the Dirac–Fock–Podolsky Lagrangian formalism of the electrodynamics, I have stated both the energy positivity of photon quanta and the strong Lorenz condition on the Fock space.

It is well known that the usually accepted Dirac type quantization scheme [3, 1] in quantum electrodynamics meets some mathematical problems related with the necessity to satisfy simultaneously the requirement of the local positive energy density, the Lorenz subsidiary condition, involved on the quantum states, and the condition of transversality. Moreover, the whole theory formulation must be relativistic covariant. As the corresponding photon quanta are massless, attempts to carry out the Dirac type quantization scheme to electrodynamics, being a vector field theory, encounter fundamental difficulties, resulting in meaningless expressions, at the early fiftieth of the past century there were undertaken efforts to overcome them by means of some weakly argued from the physics point of view mathematical tricks, suggested [4, 2] by Gupta and Bleuler, based both on the physically not motivated indefinite metric and the corresponding weak Lorenz subsidiary condition, imposed on the related Fock space.

In the work under regard I have also revisited the Dirac quantization scheme within the Fock space representation of quantum states applied to a modified Lagrangian description of electrodynamics. Based on this quantization reformulation, there is stated that the Lorenz subsidiary condition satisfies strongly on the whole Fock space, is time-invariant and simultaneously guaranteeing the energy density positivity of photon quanta and the related relativistic covariance.

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15. Krzysztof RADZISZEWSKI — *University of Białystok, Poland*

Affinization of algebraic structures: Lie algebras

By affinization we mean a process which converts an algebraic structure with a nullary operation (i.e., in which a chosen element plays special role) into one which has no nullary operations, but is such that by a free choice of any element it is retracted to the original structure. The prime example of this procedure is the conversion of groups into heaps.

In this talk we will show how to define a Lie bracket on an affine space to obtain a Lie affgebra. We will present the main theorem which states that to construct a Lie affgebra for a given Lie algebra we need generalized derivations and an algebra element. We will consider a particular class of derivation-type Lie affgebras and then say a few words about the classification of low-dimensional Lie affgebras. The talk is based on joint work with Tomasz Brzeziński and Ryszard Andruszkiewicz.

16. Brais RAMOS PÉREZ — *Universidade de Santiago de Compostela, Spain*

When is the Drinfeld Double Hopf algebra part of a Hopf brace structure?

Given two Hopf braces in a symmetric monoidal category C, namely $\mathbb{A} = (A_1, A_2)$ and $\mathbb{H} = (H_1, H_2)$, such that (A_1, H_1) and (A_2, H_2) form matched pairs of Hopf algebras, we investigate the conditions under which the pair

$$(A_1 \bowtie H_1,\ A_2 \bowtie H_2)$$

defines a new Hopf brace. In particular, we establish necessary and sufficient conditions for the pairs

$$(A_1 \otimes H_1,\ A_2 \bowtie H_2) \quad \text{and} \quad (A_1 \bowtie H_1,\ A_2 \sharp H_2)$$

to be Hopf brace structures, viewing them as special instances of the general problem above.

The Drinfeld Double is one of the most prominent examples of a quasitriangular Hopf algebra, typically neither commutative nor cocommutative, and therefore provides solutions to the Quantum Yang–Baxter equation. As an application of the previous constructions, we study conditions under which the Drinfeld Double can be realized as part of a Hopf brace.

17. **Stefan RAUCH** — *Linköping University, Sweden*

Transfer of integrable structures of soliton hierarchies to integrable Newton equations

In 1990-ties I have in collaboration with M. Antonowicz, P. Kulish, A. Orlov developed technique of transferring integrable structures of soliton hierarchies to finite dimensional invariant manifolds defined as stationary/restricted flows. They are systems of ODE's that, after suitable parametrisation, acquire form of Newton equations with velocity independent forces.

Structures of integrable hierarchies such as bi-hamiltonian formulation, Lax representation, Miura transformation, r-matrix give rise to corresponding structures for 2nd order ODE's being stationary and restricted flows. These structures were interesting by their own and gave rise to further, new classes of integrable ODE´s of mechanical type.

These results have been largely forgotten and the purpose of this talk is to bring them again to light as there are still many new results about integrability of Newton equations to be found there. I shall illustrate this approach by discussing 2 simple examples: the stationary KdV equation and the Garnier system of motion of a particle in a quartic potential.

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18. **Jonathan SMITH** — *Iowa State University, USA*

Quantization of web geometry

The prime motivation for this talk is the ongoing search for a "pregeometry" [1] as a spacetime geometry that is quantized at the Planck length of 10^{-38} miles. Classical web geometry is well placed to provide a background for this search. It already underlies both quantum mechanics (compare, say, Wooters' use of webs [2] in his approach to finite state Wigner functions) and general relativity (compare, say, the expression of curvature [3] within web geometry). Thus, the problem of quantizing web geometry emerges as a potentially important part of the general problem of appropriately quantizing spacetime.

Classical web geometry uses ordered pairs and projections to factors. However, in the general quantum setting of a symmetric, monoidal structure, this approach is no longer available. An alternative is needed. The points of a classical web appear as categorical points (morphisms out of the terminal object) in a suitable category of semisymmetrizations of classical quasigroups [4]. Semisymmetrization of quantum quasigroups thus presents itself as one possible approach to the quantization of web geometry [5].

We initiate this program by examining the most amenable setting, Cartesian symmetric monoidal categories of modules over a commutative, unital ring. We determine linear quantum quasigroup structures that provide comultiplications to extend the semisymmetrization multiplication of a linear quasigroup. In particular, if the linear quasigroup structure comes from a real or complex affine plane, a complete classification of the quantum semisymmetric comultiplications is provided, based on the solution of a system of cubic equations.

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19. **Marzena SZAJEWSKA** — *University of Białystok, Poland*

Quadratic deformations and 2-compatible Lie algebras

In this talk we discuss quadratic deformations of Lie algebras and the associated notion of a 2-compatible Lie algebra. After reviewing the correspondence between quadratic deformations and compatibility conditions on bilinear maps, we introduce genuinely quadratic structures, for which the second-order term of the deformation is essential. We present examples of such structures and describe the classification of all 3-dimensional non-contact genuinely quadratic 2-compatible Lie algebras up to isomorphism. Possible connections with contact Lie algebras and further directions of research will also be discussed.

It is a joint work with Severin Pošta.

20. **Vojtěch TESKA** — *Czech Technical University in Prague, Czech Republic*

Splitting of discrete Weyl orbit function transforms in the root system A_3

Irreducible crystallographic root systems and their corresponding Weyl groups determine invariant lattices which serve as the basis of discrete Fourier-like transforms. A finite fragment of a shifted invariant lattice located in the fundamental domain of a generalized affine Weyl group plays the role of the time domain signal while the finite frequency basis consists of Weyl orbit functions labeled by elements of the weight lattice. Recently developed general theory of these transforms is used to decompose the original point set into smaller subsets, called splitting point sets, which allow a different transform to be carried out. The relationship between corresponding Fourier coefficients will be described and an example in the root system A_3 will be presented.

21. **Alice Barbara TUMPACH** — *Wolfgang Pauli Institut, Austria*

Kähler and Poisson Geometry of affine coadjoint orbits of infinite-dimensional unitary groups

Motivated by the study of central extensions of infinite-dimensional unitary groups and their coadjoint orbits, we present families of affine coadjoint orbits of unitary groups that possess natural weak Kähler structures. Some of these orbits possess also a (non-symplectic) Poisson structure inherited form the Poisson–Lie group structure of the corresponding unitary groups. The restricted Grassmannians modeled on Schatten classes L^p will serves as our running example in order to illustrate the different cases depending on p.

This talk is based on joined works with T. Goliński, G. Larotonda and A.Tahiri.

22. **Krzysztof ZAJKOWSKI** — *University of Białystok, Poland*

Gaussian vs α-Gaussian world

Gaussian distributions occupy a central place in probability theory. Their role in the statistical study of physical phenomena is invaluable. This talk will present the concept of α-Gaussian distributions. Examples will demonstrate their applications to the study of processes (phenomena) occurring in the world around us.

23. **Marco ZAOPO** — *Milano, Italy*

Group representations of Lorentz transformations extended to superluminal observers

We construct an extension of the proper orthochronous Lorentz group that includes space-time transformations for observers moving with superluminal relative velocities in arbitrary direction. This extension is generated by a realization of the Klein four group depending on polar and azimuthal angles identifying a spatial direction and is obtained with matrices representing infinite velocity limits of superluminal Lorentz boosts. The resulting group has the same identity component of the whole Lorentz group $O(3,1)$ but involutive operators corresponding to an infinite speed boost and its negative in place of parity and time reversal. Different spatial directions in the definition of Klein group realization give rise to equivalent group extensions. We then define the extended Poincaré group including translations and classify its unitary irreducible representations (UIRs). The resulting UIRs are induced from Wigner's UIRs of standard Poincaré group and depend on the action of the extended Lorentz group defined on momentum space. UIRs corresponding to non lightlike orbits restrict to the ordinary Poincaré subgroup as a multiplicity one direct sum of a massive forward, a massive backward and a tachyonic Wigner UIR while for lightlike orbits as a multiplicity one direct sum of a forward and backward massless Wigner UIR. We then derive wave equations corresponding to solutions of the Casimir eigenvalue problem of Poincaré algebra obtained differentiating the above representations. This set of equations contains all the wave equations known to date in quantum field theory together with new wave equations describing tachyonic behaviour and a new class of massless representations. We finally show that tachyonic wave functions provide a relevant representation theoretic tool for interpretation of parity violation phenomena in quantum field theory.

Virtual contributed lectures

1. **Kyriaki-Evangelia ASLANI** — *University of the Peloponnese, Greece*

A multivector generator of time evolution in the phase plane

A geometric formulation of time evolution is proposed in which the time derivative is represented by a multivector generator acting on the phase plane within two-dimensional geometric (Clifford) algebra. Rather than extending time as an additional scalar or complex parameter, the approach focuses on the algebraic structure of the generator responsible for dynamical evolution. Using the isomorphism between linear phase-space generators and multivectors in $Cl(2,0)$, the time evolution operator is shown to admit a natural decomposition into scalar, vector, and bivector components, each associated with a distinct geometric action. The bivector part generates symplectic, Hamiltonian evolution and preserves the oriented phase-space area, while the scalar part produces uniform contraction or expansion and provides a direct geometric characterization of irreversibility. Vector components generate reversible but anti-symplectic transformations, corresponding to reflections that reverse the orientation of the symplectic form while preserving its magnitude. The general solution of linear systems follows from the exponential of the multivector generator and admits a transparent geometric interpretation. The reversible sector is naturally classified into elliptic, hyperbolic, and nilpotent cases, yielding oscillatory, overdamped, and critically damped behavior without invoking ad hoc assumptions. Within this framework, complex time emerges only as a restricted case corresponding to the even subalgebra; in the general setting, time evolution is intrinsically multivectorial. The proposed formulation provides a unified geometric language for reversible and irreversible dynamics and highlights the role of generator structure, rather than time itself, in shaping temporal evolution.

2. **Luca CAMPOBASSO** — *Łódź University of Technology, Poland*

Scattering on a rectangular potential barrier in phase space quantum mechanics

We present a treatment of quantum scattering on a rectangular potential barrier within the phase-space formulation of quantum mechanics. We present a treatment of quantum scattering on a rectangular potential barrier within the phase-space formulation of quantum mechanics. The Wigner energy eigenfunction is constructed directly in phase space without recourse to the Hilbert-space wave function. The solution is built from a structured decomposition of the Wigner function, and it is shown that standard physical requirements are sufficient to uniquely determine all parameters up to the incident flux.

3. **Adrian CHITAN** — *Western University, Canada*

   Stratification of the Jeffrey–Weitsman half-density quantization of the Moduli Space of flat connections and Witten–Chern–Simons invariants

   We present a novel reconstruction of the Jeffrey–Weitsman–Witten invariant through a stratified half-density quantization of the moduli space of flat connections, providing a rigorous and extended definition applicable to closed oriented 3-manifolds. Unlike previous approaches that rely solely on smooth components, this framework incorporates the entirety of the moduli space by leveraging the smooth stratified structure of the Lagrangian leaf. By defining a global volume form and a covariant constant half-density on the tangent spaces of the smooth strata, we naturally employ a reduction of the structure group at reducible connections. This construction is embellished with the level k weighting conjectured by Freed and Gompf, where the $k^{(h^1-h^0)/2}$ scaling factor arises as a consequence. We demonstrate that the resulting stratified BKS pairing exactly reproduces the topological invariant across all classes of flat connections, thereby extending and fortifying the invariant of Jeffrey–Weitsman and more closely approaching that of Witten's conjecture. In this talk we discuss this stratification and consider examples which showcase the refinement.

4. **Miroslav KORBELÁŘ** — *Czech Technical University in Prague, Czech Republic*

   Structure of Clifford groups of composite finite quantum systems

   We consider the Clifford groups of general multipartite quantum systems with configuration space $\mathbb{Z}_{n_1} \oplus \cdots \oplus \mathbb{Z}_{n_k}$. It is known that the Clifford group is a natural semidirect product provided the dimension $N = n_1 \cdots n_k$ of the corresponding Hilbert space is an odd number.

   For even N special results on the Clifford groups are scattered in the mathematical literature, but they do not concern the semidirect product structure. We show that for even $N = n_1 \cdots n_k$ both Clifford group and the projective Clifford group are natural semidirect products if and only if N is not divisible by four. Our approach is based on Based on relation of generators of the associated symplectic group $\mathrm{Sp}_{[n_1,\ldots,n_k]}$.

   (a joint work with J. Tolar)

5. **Kaushlendra KUMAR** — *Queen Mary University of London, United Kingdom*

   Scalar-anchored Connes distance and Helstrom geometry of the qubit Bloch ball

   Trace distance gives the operational distinguishability of two qubit states in equal-prior Helstrom discrimination. This talk describes a finite scalar-qubit-scalar spectral model in which this trace geometry is obtained from Connes distance, up to a single calibration factor. The algebra is $\mathbb{C}_L \oplus M_2(\mathbb{C}) \oplus \mathbb{C}_R$, with scalar reference sectors coupled to the qubit block by isotropic identity links. On the middle qubit sector, the spectral distance is the usual trace distance multiplied by a scale fixed by the two Dirac couplings. Hence the full Bloch ball, including mixed states, inherits its standard chordal trace-distance geometry from the finite spectral metric. Once the scale is known, the same distance gives the Helstrom success and error probabilities. The scalar sectors play a separate role. Their distances are calibration lengths for the finite geometry, not additional qubit discrimination probabilities, and they provide a scalar-scalar consistency check for the two links.

6. **Patryk MIESZKALSKI** — *Uniwersytet Wrocławski, Polska*

   Differential structure on quantum spacetime and its quantum symmetry group

7. **Oscar ROSAS-ORTIZ** — *CINVESTAV, Mexico*

Optimized two-qubit systems

We investigate the entanglement properties of two-qubit systems by using the minimum of essential parameters. The idea is to construct density operators whose reduced one-qubit states share the same entropy, regardless of whether the state of the entire system is pure or mixed. We show that the parameters so found define a convex set S that can be divided in subsets whose points are linked to separable states and regions where entanglement may be found. Geometrically, S is nothing more than a right-triangle in $R \times R$.

8. **Amin TAHIRI** — *Utrecht University, Netherlands*

Dressing actions and symplectic leaves of Banach Poisson–Lie groups

It is well known that, in finite dimensions, the characteristic distribution associated with a Poisson manifold is integrable and that its integral leaves carry natural symplectic structures, yielding the symplectic foliation of the manifold. In the setting of Poisson–Lie groups, Semenov-Tian-Shansky showed in [1] that the orbits of the right dressing action coincide with the symplectic leaves of the Poisson structure. The purpose of this talk is to present an analogous result in the Banach setting, following the approach of Lu and Weinstein in [2] adapted to Banach Poisson–Lie groups as defined in [3].

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[2] J.-H. Lu, A. Weinstein, Poisson Lie groups, dressing transformations, and Bruhat decompositions, J. Diff. Geom. 31/2, 1990

[3] A.B. Tumpach, Banach Poisson–Lie Groups and Bruhat–Poisson Structure of the Restricted Grassmannian, Comm. Math. Phys. 373, 2020

Poster presentations

1. **Grzegorz JAKIMOWICZ** — *University of Białystok, Poland*

Cyclic Lie–Rinehart Algebras

We study Lie–Rinehart algebra structures in a framework provided by duality pairings of modules over unital commutative associative algebra. Thus, we construct new examples of Lie brackets corresponding to a fixed anchor map whose image is a cyclic submodule of the derivation module, and therefore we call them cyclic Lie–Rinehart algebras. Special cases of our construction include Lie algebroid structures on cotangent bundles of differential manifolds and also certain differential operators that occur in mathematical physics.

2. **Alina KOŚCIUKIEWICZ** — *University of Białystok, Poland*

Unique trace property for reduced L^p-group algebras

A trace on a Banach algebra A is a unital contractive functional $\tau : A \to \mathbb{C}$, such that $\tau(ab) = \tau(ba)$ for all $a, b \in A$. If there is exactly one trace on A, we say that A has the unique trace. The L^p-group algebra is the generalisation of a C^*-algebra. We obtain it by the same construction, but instead of representing the Banach algebra $\ell^1(G)$ on the Hilbert space $\ell^2(G)$, we represent it on its "tilted" version $\ell^p(G)$ for all $p \in [1, \infty]$. In the poster, I present theorems concerning the uniqueness of the trace of L^p-group algebras and prove a lemma by means of which we obtain the unique trace characterisation.

3. **Petr NOVOTNÝ —** *Czech Technical University in Prague, Czech Republic*

   Standard Manin triples and corresponding Drinfeld doubles in dimension $4+4$

   Drinfeld doubles and Manin triples have been systematically used in Poisson–Lie T-plurality, integrable systems, string backgrounds, and generalized geometry. The classification of these structures is known up to dimension $3+3$. In higher dimensions, there are only isolated examples in the literature. We present a classification of a certain class of so-called standard Manin triples in dimension $4+4$.

4. **Severin POŠTA —** *Czech Technical University in Prague, Czech Republic*

   Generating function for orthogonal polynomials of (anti)symmetric multivariate sine functions of three variables

   The generating functions of four kinds of orthogonal polynomials of (anti)symmetric multivariate sine functions of three variables are computed. The generating functions hand as a powerful tool for investigating symmetries, parity relations and efficient computer implementation and handling of the polynomials. The recurrence relations algorithms for the calculation of the polynomials are superseded by explicit evaluation formulas which can be derived from the generating functions.

5. **Aneta SLIŻEWSKA —** *University of Białystok, Poland*

   Groupoid of partial isometries over the restricted Grassmannian

   Partial isometries in a W^*-algebra form a groupoid, which additionally possesses a structure of Banach Lie groupoid. It is possilbe to construct the differential structure on the set of partial isometries over the restricted Grassmannian.

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